# Public Economics (ECON 131) Section \#3: Tax Incidence and Efficiency Costs 

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## 1 Tax Incidence

### 1.1 Key results to keep in mind

1. The statutory incidence of a tax is not generally equal to its economic incidence.
2. The equilibrium outcome is independent of who nominally pays the tax; it does not matter if the statutory incidence is on the buyer or the seller.
3. The more inelastic side bears more of the tax burden.

### 1.2 Formulas

- Start with the following:
- Demand for good $x$ is $D\left(p^{\prime}\right)$ decreases with $p^{\prime}=p+t$, where $p^{\prime}$ is the price including tax that the consumers pay.
- Supply for good $x$ is $S(p)$ increases with $p$, where $p$ is the price received by suppliers.
- Equilibrium condition: $Q=S(p)=D(p+t)$
- Starting from $t=0$ and $S(p)=D(p)$, we want to characterize $d p / d t$ : the effect of a tax increase on price, which determines who bears the effective burden of a tax:
- A small change in the tax, $d t$, generates a small price change, $d p$, so that equilibrium holds:

$$
\begin{aligned}
S(p+d p) & =D(p+d p+d t) \\
S(p)+S^{\prime}(p) d p & =D(p)+D^{\prime}(p)(d p+d t) \\
S^{\prime}(p) d p & =D^{\prime}(p)(d p+d t) \\
\frac{d p}{d t} & =\frac{D^{\prime}(p)}{S^{\prime}(p)-D^{\prime}(p)}
\end{aligned}
$$

- Use the following useful definitions:
- Elasticity: Percentage change in quantity when price changes by one percent. We often use elasticities in economic analysis because they are unit-free.
- Price elasticity of demand: $\varepsilon_{D}=\frac{p^{\prime}}{D} \frac{d D}{d p^{\prime}}=\frac{p^{\prime} D^{\prime}\left(p^{\prime}\right)}{D\left(p^{\prime}\right)}<0$
- Price elasticity of supply: $\varepsilon_{S}=\frac{p}{S} \frac{d S}{d p}=\frac{p S^{\prime}(p)}{S(p)}>0$
- Then, we have the following formulas summarizing responses to taxes:

$$
\begin{aligned}
\frac{d p}{d t} & =\frac{D^{\prime}(p)}{S^{\prime}(p)-D^{\prime}(p)}=\frac{\varepsilon_{D}}{\varepsilon_{S}-\varepsilon_{D}} \\
\frac{d p^{\prime}}{d t} & =\frac{\varepsilon_{S}}{\varepsilon_{S}-\varepsilon_{D}}
\end{aligned}
$$

- Also, $-1 \leq \frac{d p}{d t} \leq 0$ and $0 \leq \frac{d p^{\prime}}{d t}=1+\frac{d p}{d t} \leq 1$.
- When do consumers bear the entire burden of the tax? $\left(d p / d t=0\right.$ and $\left.d p^{\prime} / d t=1\right)$

1. $\varepsilon_{D}=0$ [inelastic demand]

- Example: short-run demand for gas inelastic (need to drive to work)

2. $\varepsilon_{S}=\infty$ [perfectly elastic supply]

- Example: perfectly competitive industry
- When do producers bear the entire burden of the tax? $\left(d p / d t=-1\right.$ and $\left.d p^{\prime} / d t=0\right)$

1. $\varepsilon_{S}=0$ [inelastic supply]

- Example: fixed quantity supplied (land)

2. $\varepsilon_{D}=-\infty$ [perfectly elastic demand]

- Example: there is a close substitute, and demand shifts to this substitute if price changes.


### 1.3 Practice problems

### 1.3.1 Gruber, Ch.19, Q. 2 (modified)

The demand for rutabagas is $Q=2,000-100 P$ and the supply of rutabagas is $Q=-100+200 P$.

1. Who bears the statutory incidence of a $\$ 2$ per unit sales tax on rutabagas, paid by the producer and reflected in the sticker price?
2. Who bears the economic incidence of this tax? (Discuss)
3. What is the equilibrium price without the tax?
4. What is the equilibrium price with the tax?
5. What are the tax burdens on the consumer and producer?

## Solution:

1. If the tax is on the sale of rutabagas, the buyer bears the statutory incidence, since the "sticker price" of rutabagas includes the tax.
2. Economic incidence is determined by relative elasticities. As we will see, the quantity supplied is more responsive to a change in price, so the less elastic consumers will bear most of the economic incidence. To calculate the relative burdens, solve the equilibrium condition with and without the tax.
3. Without the tax: $2,000-100 P=-100+200 P$. Price $=\$ 7.00$.
4. With the tax, the price the supplier receives is reduced by $\$ 2.00$. The equilibrium condition:

$$
\begin{aligned}
2,000-100 P & =200(P-2)-100 \\
2,000-100 P & =200 P-500 \\
2,500 & =300 P \\
P & =8.33
\end{aligned}
$$

Thus price $=\$ 8.33$.
5. The consumers' tax burden = (posttax price - pretax price $)+$ tax payments by consumers, here $\$ 8.33-\$ 7.00+0 \approx \$ 1.33$.
The producers' tax burden = (pretax price - posttax price) + tax payments by producers, here $\$ 7.00-\$ 8.33+\$ 2.00 \approx \$ 0.67$. In this case the consumer bears a larger share of the tax burden than the producer.

### 1.3.2 Gruber, Ch.19, Q. 3 (modified)

The demand for rutabagas is still $Q=2,000-100 P$ and the supply is still $Q=-100+200 P$, as in Question 2. Governor Sloop decides that instead of imposing the $\$ 2$ sales tax described in Question 2, the government will instead force consumers to pay the tax, such that the tax is no longer reflected in the sticker price. What will happen to the "sticker price" on rutabagas? How will the size of the consumer tax burden change?

## Solution:

- The sticker price for consumers when they bear the statutory burden of the $\$ 2$ tax is $P \approx \$ 6.33$. Similarly to question 2 , this is the solution to $2,000-100(P+2)=-100+200 P$.
- Before, the sticker price for consumers when firms pay the tax is the solution to $2,000-100 P^{\prime}=$ $-100+200\left(P^{\prime}-2\right)$, so the sticker price in that case is $P^{\prime} \approx \$ 8.33$, or $\$ 2.00$ more than the sticker price when consumers pay the tax.
- Consumers pay exactly the same net amount as before: now, they pay the $\$ 6.33$ sticker price plus a $\$ 2$ tax, and before they paid $\$ 8.33$ directly. The economic incidence of the tax is unchanged.


### 1.3.3 Gruber, Ch.19, Q. 4

The demand for football tickets is $Q=360-10 P$ and the supply of football tickets is $Q=20 P$. Calculate the gross price paid by consumers after a per-ticket tax of $\$ 4$. Calculate the after-tax price received by ticket sellers.

## Solution:

- For this answer, it does not matter whether the tax is added to the price paid by the consumers or subtracted from the price the sellers keep.
- Adding it to the consumers' price yields demand of $360-10(P+4)$, which is set equal to supply to yield the equilibrium after-tax price: $360-10 P-40=20 P$.
- Simplified, $320=30 P$, so base price $=\$ 10.67$ and price $+\operatorname{tax}=\$ 14.67$ paid by consumers.
- Producers keep only $\$ 14.67$ - $\$ 4=\$ 10.67$.
- Without the tax, $360-10 P=20 P \Rightarrow 360=30 P$; the price paid by consumers and kept by sellers is $\$ 12$.


## 2 Efficiency Costs of Taxation

- We know that governments use tax revenue to finance public goods or to redistribute income from rich to poor. As you saw in lecture if the government could do lump-sum taxes and transfers based on earning ability then any redistributive equilibrium could be reached.
- However, in practice earning ability is hard to observe: The government does not know if a person's income is low because they cannot work (e.g disability) or because they do not want to work. Therefore the government taxes observed outcome such as income and consumption.
- As a consequence tax revenue generally has an efficiency cost: to generate $\$ 1$ of revenue, need to reduce welfare of the taxed individuals by more than $\$ 1$. Efficiency costs come from distortion of behavior.
- Deadweight burden (also called excess burden) of taxation is defined as the welfare loss (measured in dollars) created by a tax over and above the tax revenue generated by the tax.
- We can measure the deadweight burden of a tax by the Harberger Triangle:

$$
D W B=\frac{1}{2} d Q \cdot d t=\frac{1}{2} \cdot \frac{\varepsilon_{S} \cdot \varepsilon_{D}}{\varepsilon_{S}-\varepsilon_{D}} \cdot \frac{Q}{p}(d t)^{2}
$$

- In a simple supply and demand diagram, welfare is measured by the sum of the consumer surplus and producer surplus. We can measure the welfare loss of taxation as the change in consumer + producer surplus minus tax collected: it is the triangle on the figure.

- The inefficiency of any tax is determined by the extent to which consumers and producers change their behavior to avoid the tax; deadweight loss is caused by individuals and firms making inefficient consumption and production choices in order to avoid taxation.
- If there is no change in quantities consumed, the tax has no efficiency costs.


### 2.1 Practice problems

### 2.1.1 Gruber, Ch.20, Q. 2

The government of Washlovia wants to impose a tax on clothes dryers. In East Washlovia the demand elasticity for clothes dryers is -2.4 while in West Washlovia the demand elasticity is -1.7 . Where will the tax inefficiency be greater? Explain.

## Solution:

- The more elastic the demand, the more inefficient the tax; therefore, the tax will be less efficient in East Washlovia.
- Tax inefficiency is measured by the area of deadweight loss it generates.
- The base of the deadweight loss triangle will be the same in both areas, as it is the dollar amount of the tax.
- The height of the deadweight loss triangle, though, will differ, because it is the quantity change, or the quantity response, to the tax.
- Elasticity is a measure of quantity response to a price change: the higher the elasticity, the greater the quantity change for a given price change. When demand is elastic, a price change will distort quantity demanded by relatively more than when it is inelastic, and it is this quantity distortion that causes inefficiency.


### 2.1.2 Gruber, Ch.20, Q. 10 (adapted)

The market demand for pet turtles is $Q=2,600-20 P$, and the government intends to place a $\$ 4$ per turtle tax on pet turtle purchases. Calculate the deadweight loss of this tax when:

1. Supply of pet turtles is $Q=400$.
2. Supply of pet turtles is $Q=12 P$.
3. Explain why the deadweight loss calculations differ between 1 and 2.

## Solution:

1. The quantity before the tax is 400 ; the quantity after the tax is 400 . When supply is always 400 turtles, the deadweight loss of the turtle tax is $\frac{1}{2} \cdot(4 * 0)$, or 0 . There is no change in supply, so there is no deadweight loss.
2. In this case, supply is not completely inelastic, so before-tax and after-tax quantities must be calculated.

- Before tax: $2,600-20 P=12 P \quad \Rightarrow \quad P=\$ 81.25 ; Q=12 * 81.25=975$.
- After tax:

$$
\begin{gathered}
2,600-20 P=12(P-4) \Rightarrow \\
2,600-20 P=12 P-48 \Rightarrow \\
2,648=32 P \Rightarrow \\
P=82.75 ; \quad Q=2,600-(20 * 82.75)=945
\end{gathered}
$$

- The quantity change is $975-945=30$, so the area of the deadweight loss triangle is $\frac{1}{2}(30 \cdot 4)=60$.

3. Deadweight loss is caused by changes in the equilibrium quantity. In (1), because supply was perfectly inelastic, there was no change in quantity. When quantity does not change, the tax has caused no distortion. Thus, there is no deadweight loss, only a transfer of money from the seller to the government.
